SERIAL NUMBER 2:

STATISTICS

1. Define consistent and unbiased estimators. Show that the sample mean is a consistent and unbiased estimator of the population mean.

2. Show that the sample variance is not an unbiased estimator of the population variance. Find an unbiased estimator of population variance.

3. If T is an unbiased estimator of a population parameter θ, show that T^2 is a positively biased estimator of θ^2; but if T is a consistent estimator of θ then show that T^2 is also a consistent estimator of θ^2.

4. Of 400 mangoes selected at random from a large stock, 53 were found to be bad. Test at 1% significance level the hypothesis that on the average 10% of the mangoes were bad.

5. The regression lines of X on Y and of Y on X are $x = 4y + 5$ and $y = kx + 4$ respectively. Show that $0 < k \leq \frac{1}{4}$. If $k = \frac{1}{16}$ find the means of the variable and the correlation coefficient between them.

6. In a random sample of 400 articles 40 are found to be defective. Obtain 95% confidence interval for the true proportion of defective in the population of that article.

7. Suppose $x_1, x_2, ..., x_n$ is a random sample from a normal distribution with mean μ and variance σ^2. Given that μ and σ are both unknown, find the maximum likelihood estimators (MLEs) of μ and σ. What will be the MLE of $\frac{1}{\mu}$?

8. The number of car accidents in the i-th city is x_i in a year. The number of car accidents follow Poisson distribution with mean $k_i\lambda$, where k_i is the population of the i-th city. Choose n cities randomly for a sample. Find the MLE of λ.

9. To test whether a coin is perfect, the coin is tossed five times. The null hypothesis of perfectness is rejected if more than four heads are obtained. What is the probability of Type I error? Find the probability of Type II error when the corresponding probability of getting head is 0.2.

10. A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with standard deviation $\sqrt{7.056}$. Test the hypothesis that the population mean is 69 at 1% level of significance. [Given that $P(0 < z < 2.58) = 0.495$]

11. A dice was thrown 400 times and 'six' resulted eight times. Do the data justify the hypothesis of an unbiased dice?

12. Can you fit a curve $y = ab^{c^x}$, where a, b and c are parameters? Suggest your computational procedure.

13. Test whether the following two lines are regression lines or not:
i) $2y + 5x + 7 = 0$
ii) $y + 7x + 12 = 0$

14. In Kolkata, 325 men out of 600 were found to be smokers. Does this information support the conclusion that the majority of men in Kolkata are smokers? (State the hypothesis clearly).

15. Derive normal equations to fit
i) the straight line $y = a + bx$
ii) the parabola $y = a + bx + cx^2$

16. If σ_x, σ_y and σ_{x-y} are the standard deviations of the variates x, y ans x-y respectively, then show that

$$\rho_{xy} = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$$

17. In 1950 in India the mean life expectancy was 50 years. If the life expectancies from a random sample of 11 persons are 58.2, 56.2, 54.2, 50.4, 44.2, 61.9, 57.5, 53.4, 49.7, 55.4, 57.0 does it confirm the expected view?