INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET

THIRD SEMESTER (For all Engineering Branches)

SUBJECT: MATHEMATICS-III

SUBJECT CODE: MA 301

SERIAL NUMBER 3:

LAPLACE TRANSFORM

Find the Laplace Transforms of the following functions:

(1) $(\sin x - \cos x)^2$ Ans. $(s^2 - 2s + 4)/s(s^2 + 4)$

(2) $x^3 e^{-3x}$ Ans. $6/(s+3)^4$

(3) $2e^{3x} \sin 4x$ Ans. $8/(s^2 - 6s + 25)$

(4) $(x+1)^2 e^x$ Ans. $s^2 + 1/(s-1)^3$

(5) $e^{-x} sin^2 x$ Ans. $2/(s+1)(s^2 + 2s + 5)$

(6) $4 \sinh 5x - 3 \cosh 4x$ Ans. $(20 - 3s)/(s^2 - 25)$

(7) $f(x) = \sin x, 0 < x < \pi$ Ans. $(e^{-s\pi} + 1)/(s^2 + 1)$

$= 0 \ , \ x > \pi$

(8) $f(x) = \mathrm{e^x} \ , \ 0 < x < 5$ Ans. $(1 - e^{-5(s-1)})/(s-1) + 3e^{-5s}/5$

$= 3 \ , \ x > 5$

(9) $f(x) = \mathrm{e}^{2(\mathrm{x}-3)} \sin 3(\mathrm{x} - 3) \ , \ x > 3$ Ans. $3\, e^{-3s}/(s^2 - 4s + 13)$

$= 0 \ , \ x < 3$

(10) $(\cos ax - \cos bx)/x$ Ans. $\frac{1}{2} log(\frac{s^2+b^2}{s^2+a^2})$

(11) If $\mathrm{L}\{\varphi(x)\} = f(s)\ find\ \mathrm{L}\{\varphi(x) \cos kx\}$ Ans. $\{f(s - ik)\}/2 + \{f(s + ik)\}$

(12) If $\mathrm{L}\{(f(x)\} = e^{-1/s}/s$, then prove that $L\{e^{-x} f(4x)\} = e^{-4/(s+1)}/(s+1)$.

(13) If $f(x) = 2x \ , \ 0 \le x \le 1$

$= x \ , \ x > 1$

find (i) $\mathrm{L}\{(f(x)\}$ (ii) $L\{f'(x)\}$ (iii) does the result $L\{f'(x)\} = sf(x)\} - f(0)$ hold for this case. Ans. $(i)\ 2/s^2 - e^{-s}/s - e^{-s}/s^2$. $(ii)\ 2/s - e^{-s}/s$.

(14) If $L\{f''(x)\} = tan^{-1}(1/s)\ , f(0) = 2\ ,\ f'(0) = -1\ , find\ \mathrm{L}\{(f(x)\}$.

Ans. $\{2s + tan^{-1}(1/s) - 1\}/s^2$.

(15) Prove that $L\{(\sin x)/x\} = tan^{-1}(1/s)$. Hence find $L\{(\sin a\, x)/x\}$.

Ans. $tan^{-1}(a/s)$

Evaluate the following integrals

(16) $\int_0^\infty \frac{e^{-3x} - e^{-6x}}{x} dx$ Ans. $log2$

(17) $\int_0^\infty \frac{1 - cosx}{x^2} dx$ Ans. $\pi/2$

(18) $\int_0^\infty xsinxe^{-3x}\, dx$ Ans. $3/50$

(19) Let $f(x) = x\ ,\ 0 < x < \pi$

$= \pi - x\ ,\ \pi < x < 2\pi$

where $f(x) = f(2\pi + x)\ , find\ \mathrm{L}\{(f(x)\}$. Ans. $\{1 - e^{-\pi s}(\pi s + 1)\}/(1 + e^{-\pi s})s^2$.

(20) Given $f(x) = sinx\ ,\ 0 < x < \pi$

$= 0\ ,\ \pi < x < 2\pi$

$find\ \mathrm{L}\{(f(x)\}\ , where\ f(x) is\ a\ periodic\ function\ of\ period\ 2\pi.$

Ans. $1/(1 - e^{-\pi s})(s^2 + 1)$.

Evaluate the following problems

(21) $L \int_0^x \frac{\sin u}{u} du$ Ans. $\frac{1}{s} tan^{-1} \frac{1}{s}$

(22) $L\{x \int_0^x \frac{\sin u}{u} du\}$ Ans. $1/s\,(s^2 + 1) + \{cot^{-1}s\}/s^2$.

(23) $L \int_0^x \frac{\mathrm{e}^{2u} \sin u}{u} du$ Ans. $\{cot^{-1}(s - 2)\}/s$.

(24) $L \int_0^x u\, \mathrm{e}^{-3u} \cos 4u\, du$ Ans. $\frac{(s^2+6s-7)}{s(s^2+6s+25)^2}$.

Evaluate the following

(25) $L^{-1}\{6s/(s^2 - 16)\}$ Ans. $6cosh4x$.

(26) $L^{-1}\{(4s + 12)/(s^2 + 8s + 16)\}$ Ans. $4e^{-4x}(1 - x)$.

(27) $L^{-1}\{(s^2/(s + 2)^3\}$ Ans. $e^{-2x}(1 - 4x + 2x^2)$.

(28) $L^{-1}\{e^{-2s}/s^2\}$ Ans. $(x - 2)\ for\ x > 2\ and\ 0\ for\ x < 2$.

(29) $L^{-1}\{s\, e^{-2s}/(s^2 + 3s + 2)\}$ Ans. $2e^{-2x+4} - e^{-x+2}\ for\ x > 2\ and\ 0\ for\ x < 2$.

(30) $L^{-1}\{s/(s^2-16)^2\}$ Ans. $\sinh 4x/8$

(31) $L^{-1}\{\log(1+1/s^2)\}$ Ans. $2(1-\cos x)/x$.

(32) $L^{-1}[log\{(s+2)/(s+1)\}]$ Ans. $(e^{-x}-e^{-2x})/x$

(33) $L^{-1}\{tan^{-1}(s+1)\}$ Ans. $-e^{-x}\sin x/x$

(34) $L^{-1}[\log\{(s-1)/(s+1)\}^{1/2}]$ Ans. $-\sinh x/x$

(35) $L^{-1}\{(s+2)/(s+3)s^2\}$ Ans. $2x/3-e^{-3x}/9+1/9$.

(36) $L^{-1}[\log\{(s^2+1)/(s^2+s)\}]$ Ans. $(1+e^{-x}-2\cos x)/x$

(37) $L^{-1}\{1/(s-1)(s-2)\}$ Ans. $e^{-x}-e^{-2x}$

(38) $L^{-1}\{1/(s+1)(s^2+1)\}$ Ans. $(\sin x+e^{-x}-\cos x)/2$

(39) $L^{-1}\{1/s(s^2+4)^2\}$ Ans. $(1-x\sin 2x-\cos 2x)/16$

(40) Using convolution theorem verify that $\int_0^u \sin x\cos(u-x)\,dx = usin\,u/2$.

SERIAL NUMBER 4:

LINEAR PROGRAMMING PROBLEMS

1. If α_1 = (1,2,0), α_2 = (3,-1,1), α_3 = (4,1,1);show that the set S= $\{\alpha_1,\alpha_2,\alpha_3\}$ is linearly dependent.
2. Show that the set of vectors S = {(1,2,4),(1,0,0),(0,1,0),(0,0,1)} is linearly dependent in R^3 but the set T = {(1,2,0),(0,3,1),(-1,0,1)} is linearly independent.
3. Show that the vectors α = (1,2,4), β = (2,-1,3), γ = (0,1,2), δ = (-3,7,2) are linearly dependent and the relation between them is -9α+12β+5δ-5γ=0.
4. Show that the vectors
   (i) {(2,1,0),(1,1,0),(4,2,0)} form a basis of R^3
   (ii) {(1,1,2),(1,2,5),(5,3,4)} do not form a basis of R^3
   (iii) {(2,1,4),(1,-1,2),(3,1,-2)} form a basis of R^3

5. Let V(R) = P_n be the vector space of polynomials in t over the field of real numbers of degree $\leq$ n, show that the set S = $\{1, t, t^2, \ldots, t^n\}$ is a basis of V.
6. Obtain all the basic solutions to the following system of linear equation

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$

(Ans : $(2,1,0)^T, (5,0,-1)^T, (0,\frac{5}{3},\frac{2}{3})^T$)

7. Show that the following system of linear equations has two degenerate basic feasible solutions and the non-degenerate basic solution is not feasible:

$$3x_1 + x_2 - x_3 = 3$$
$$2x_1 - 2x_2 + x_3 = 2$$

8. Show that although $(2,3,2)^T$ is a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$
$$x_1, x_2, x_3 \geq 0,$$

It is not a basic solution. How many basic solutions this system may have ? Find all the basic feasible solution of the system.

9. Show that all three of the basic solutions of the system

$$x_1 + 2x_2 + 3x_3 = 6$$

$$2x_1 + x_2 + 4x_3 = 4$$

Exists and they are $(0^*, \frac{12}{5}, \frac{2}{5})^T, (-6, 0^*, 4)^T$ and $(\frac{2}{3}, \frac{8}{3}, 0^*)^T$.

10. If x_1, x_2 be real ,show that the set given by
    (i) $S_1 = \{(x_1, x_2): x_1+x_2 \leq 50,\ x_1+2x_2 \leq 80,\ 2x_1+x_2 \geq 0,\ x_1, x_2 \geq 0\}$ is a convex set.
    (ii) $S_2 = \{(x_1, x_2): x_1, x_2 \leq 1,\ x_1 x_2 \geq 0\}$ is not a convex set.
    (iii) $S_3 = \{(x_1, x_2): x_2^2 \geq 4x_1\}$ is not a convex set.
    (iv) $S_4 = \{(x_1, x_2): x_2 - 3 \geq -x_1^2, x_1, x_2 \geq 0\}$ is a convex set.

11. Find the extreme points, if any, of the following sets :
    (i) A = $\{(x, y) : x^2 + y^2 \leq 25\}$
    (ii) B = $\{(x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1\}$
    (iii) C = $\{(x_1, x_2) : x_1^2 + x_2^2 \leq 1, x_1, x_2 \geq 0\}$

    [Ans :

    (i) Every points on the circumference of circle

    (ii) (1,1), (-1,1), (1, -1), (-1, -1)

    (iv) (0,1), (0,0), (1,0) and all points on circular boundary]

12. A production planar in a soft drink plant has two bottling machines A and B. A is designed for 8- ounce bottles and B for 16- ounce bottles. However, each can be used on both types with some loss of efficiency. The following data is available

Machine	8- ounce bottles	16- ounce bottles
A	100/ min	45/ min
B	60/ min	75/ min

The machine can be run 8 hours per day, 5 days per week, profit on 8- ounce bottle is 15 paise and on 16- ounce bottle is 25 paise. Weekly production of the drink cannot exceed

300,000 ounces and the market can absorb 25,000 of 8- ounce bottles and 7000 of 16-ounce bottles per week. The planner wishes to maximize his profit. Formulate this an model.

Ans :

Maximize Z = 0.15 x_1 + 0.25x_2

Subject to 2x_1 + 5x_2 $\leq$ 480000

5x_1 + 4x_2 $\leq$ 720000

8x_1 + 16x_2 $\leq$ 300000

$0 \leq x_1 \leq 25000$

$0 \leq x_2 = 7000$

13. A firm produces an alloy having the following specifications (i) Specific gravity $\leq$ 0.98, (ii) Chromium $\geq$ 8%, (iii) Melting point $\geq$ 450° C.

Raw materials A, B, C having the properties in the table below can be used to make the alloy.

Property	Properties of raw materials		
	A	B	C
Specific gravity	0.92	0.97	1.04
Chromium	7%	13%	16%
Melting Point	440℃	490℃	480℃

Costs of various raw materials per unit ton are : Rupees 90 for A, Rupees 280 for B and Rupees 40 for C. The objective is to find the proportions in which A, B, C be used to obtain an alloy of desired properties in which the cost of raw materials is minimum. Formulate this blending problem as an LP model.

[Ans: Minimize Z= 90x_1 + 280x_2 + 40x_3

Subject to 0.92x_1 + 0.97x_2 + 1.04x_3 $\geq$ 0.98

7x_1 + 13x_2 + 16x_3 $\geq$ 8

440x_1 + 490x_2 + 480x_3 $\geq$ 450

x_1 + x_2 + x_3 = 100

$x_1, x_2, x_3 \geq 0$

14. A firm manufactures two products A and B on which the profit earned per unit are rupees 3 and rupees 4 respectively. Each product is processed on two machines M_1 and M_2 . Product A requires one minute on M_1 and one minute on M_2 while processing of product B requires one minute on M_1 and one minute on M_2. M_1 is not available for more than 7 hours 30 minutes while M_2 is available for 10 hours during any working day. Find the number of units of products A and B need to manufactured to get maximum profit. Formulate this as an LP model and solve by graphical method.

[Ans : x_A = 0, x_B = 450, Z_{max} = 1800]

15. Using graphical method, show that the LPP

$$\text{Maximize } Z = 2x_1 + 3x_2$$
$$\text{Subject to } x_1 - x_2 \leq 2$$
$$x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Has an unbounded solution.

16. A firm manufactures three product A, B and C. The profits per unit product are rupees 3, rupees 2 and rupees 4 respectively. The firm has two machines and the required processing time in minutes for each machine on each product is given below.

Machines	Products		
	A	B	C
X	4	3	5
Y	2	2	4

Machines X and Y have 2000 and 1500 machine-minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up an LP model to maximize the profit.

[Ans. Maximize Z = $3x_A + 2x_B + 2x_C$

Subject to $4x_A + 3x_B + 5x_C \leq 2000$

$2x_A + 3x_B + 5x_C \leq 1500$

$100 \leq x_A \leq 150$, $x_B \geq 200$, $x_C \geq 50$=50.]

17. The manager of an oil refinery has to decide upon the optimal mix of two possible blending processes of which the inputs and outputs per production run as follows:

Process	Input		Output	
	Crude A	Crude B	Gas X	Gas Y
1	5	3	5	8
2	2	4	5	4

The maximum amount available of Crudes A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gas X and 80 units of gas Y must be produced. The profit per production run from process 1 and process 2 are rupees 3 and rupees 4 respectively. Formulate the problem as an LPP.

[Ans: Maximize Z = $3x_1 + 4x_2$
Subject to $5x_1 + 4x_2 \leq 200$
$3x_1 + 5x_2 \leq 150$
$5x_1 + 4x_2 \geq 100$
$8x_1 + 4x_2 \geq 80$
$x_1, x_2 \geq 0$.

18. A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm . Because of the need to ensure certain nutrient constituents, it is necessary to buy additional products, which we shall call A and B. The nutrient constituents (vitamins and proteins) in each unit of the products are given below:

Nutrients	Nutrient contents in the products		Minimum amount of Nutrients
	A	B	
1	36	6	108
2	3	12	36
3	20	20	100

Products A costs Rupees 20 per unit and product B costs Rupees 40 per unit. Formulate the L.P model for products A and B to be purchased at the lowest possible costs so as to provide the pigs with nutrients not less than given in the table.

[Ans: : Maximize Z = $20x_A + 40x_B$
Subject to $36x_A + 6x_B \geq 108$
$3x_A + 12x_B \geq 36$
$20x_A + 4x_B \geq 100$
$x_A, x_B \geq 0$.

19. Make a graphical representation of the set of constraints of the following LPP. Find the extreme points of the feasible region. Finally, solve the problems graphically.

(i) Maximize $Z = 20x_1 + 10x_2$

Subject to $x_1 + x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1, x_2 \geq 0$

(ii) Maximize $Z = 2x_1 + x_2$

Subject to $x_1 + x_2 \geq 5$

$$2x_1 + 3x_2 \leq 20$$
$$4x_1 + 3x_2 \leq 25$$
$$x_1, x_2 \geq 0$$

(iii) Maximize $Z = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1000$

$$x_1 + x_2 \leq 800$$
$$0 \leq x_1 \leq 400$$
$$0 \leq x_2 \leq 700$$

[Ans: (i) (5, 0), ($\frac{25}{4}$, 0), ($\frac{5}{2}$, 5), (0, $\frac{20}{3}$), (0,5), x_1^*= $\frac{25}{4}$, x_2^*= 0, Z_{max} = $\frac{25}{2}$.

(ii) (15, 0), (40, 0), (4, 18), (6, 12), x_1^*=6, x_2^*= 12, Z_{min} = 240.

(iii(0, 0), (400, 0), (400, 200), (200, 600), (100, 700), (0, 700), x_1^*=200, x_2^*=600, Z_{max} =2600.]

20. (a) Let $x_1 = 2, x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations

$$2x_1 - x_2 + 2x_3 = 2$$
$$x_1 + 4x_2 = 18$$

Reduce the given feasible solution to a basic feasible solution.

(b) If $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution of the LPP

Maximize $Z = x_1 + x_2 + 4x_3$

Subject to $2x_1 + x_2 + 4x_3 = 11$

$$3x_1 + 3x_2 + 5x_3 = 14$$
$$x_1, x_2, \; x_3 \geq 0$$

Find a basic feasible solution of the problem.

(c) Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ to the system of the equations

$$x_1 + 4x_2 - x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 8$$

To a basic feasible solution.

(d) $x_1 = 2, x_2 = 2, x_3 = 1, x_4 = 0$ is a feasible solution of the set of equations

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6$$
$$15x_1 + 3x_2 - 12x_3 + 5x_4 = 9.$$

Find out the basic feasible solutions and prove that one of them is non-degenerate and other is degenerate.

[Ans: (a) $x_2 = \frac{9}{2}, x_3 = \frac{13}{4}$ with x_1 = 0 (non-basic)

(b) $x_1 = \frac{1}{2}, x_3 = \frac{5}{2}$ with x_2 = 0 (non-basic)

(c) $x_1 = \frac{17}{5}, x_2 = \frac{2}{5}$ with x_3 = 0 (non-basic)

$x_2 = \frac{13}{7}, x_3 = \frac{17}{7}$ with x_1 = 0 (non-basic)

(d) x_1 =3, x_3 = 3, x_4 = 0 with x_2 = 0 (non-basic)

x_2 =0, x_3 = 0, x_4 = 0 with x_1 = 0 (non-basic)

(e) x_1 = 0, $x_2 = \frac{9}{2}, x_3 = \frac{29}{4}$.

21. Solve the following LPP by simplex method:

Minimize Z = $x_1 - 3x_2 + 2x_3$

Subject to
$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 - 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

[Ans: x_1^*=4, x_2^*= 5, x_3^* = 0, Z_{min} =-11]

22. Show that the following LPP as an unbounded solution:

Minimize Z = $4x_1 + x_2 + 4x_3 + 5x_4$

Subject to
$$-4x_1 + 6x_2 + 5x_3 - 4x_4 \leq 20$$
$$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$$
$$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$$
$$x_1, x_2, x_3, x_4 \geq 0$$

23. Solve the following LPP by simplex method:

(i) Minimize Z = $x_1 + 2x_2$

Subject to
$$-x_1 + 2x_2 \leq 8$$
$$x_1 + 2x_2 \leq 12$$
$$x_1 - 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

[Ans: $x_1^*=\frac{15}{2}$, $x_2^*=\frac{9}{4}$, Z_{min} =12])

(ii) Maximize Z = $3x_1 + 2x_2 + x_3$

Subject to $3x_1 + 4x_2 + 3x_3 \leq 42$

$5x_1 + 3x_3 \leq 45$

$3x_1 + 6x_2 + 2x_3 \leq 41$

$x_1, x_2, x_3 \geq 0$ [Ans: $x_1^*=\frac{3}{2}$, $x_2^*=\frac{7}{3}$, $x_3^* = 0$, $Z_{max} =\frac{55}{6}$]

24. Use Charnes' Big-M method to solve the following LPP :

(i) Minimize Z = $4x_1 + 8x_2 + 3x_3$

Subject to $x_1 + x_2 \geq 2$

$2x_1 + x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$ [Ans: $x_1^*=\frac{5}{2}$, $x_2^*=0$, $x_3^* = 0$, $Z_{min} =10$]

(ii) Minimize Z = $2x_1 - 3x_2$

Subject to $-x_1 + x_2 \geq -2$

$5x_1 + 4x_2 \geq 46$

$7x_1 + 2x_2 \geq 32$

$x_1, x_2 \geq 0$ [Ans: $x_1^*=4$, $x_2^*=2$, $Z_{min} =2$]

25. Consider the problem :

Minimize Z = $3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

By using the M –technique show that the optimal solution can include an artificial basic variable at the zero level.

26. A small scale industrialist produces four types of machines M_1, M_2, M_3 and M_4 made of steel and brass. The amount of steel and brass required for each machine and the number of man-weeks of labour required to manufacture and assemble one unit of each machine are as follows:

M_1	M_2	M_3	M_4	Availability

Steel	6	5	3	2	100 kg
Brass	3	4	9	2	75 kg
Man-Weeks	1	2	1	2	20

The labour is restricted to 20 man-weeks, steel is restricted to 100 kg per week and brass 75 kg per weeks. The industrialist's profit on each unit of M_1, M_2, M_3, M_4 is rupees 6,rupees 4,rupees 7 and rupees 5 respectively. How many of each type of machine should be produced to maximize his profit, and how much is his profit?

[Ans: M_1^* = 15, M_2^* = 0, M_3^* = $\frac{10}{3}$, M_4^* = 0, Z_{max} = $\frac{340}{3}$.

27. Solve the following LPP, using two-phases of the simplex method:

(i) Minimize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$ [Ans: x_1^*=$\frac{21}{13}$, x_2^*=$\frac{10}{13}$, Z_{min} =$\frac{31}{13}$]

(ii) Minimize $Z = 2x_1 + x_2$

Subject to $5x_1 + 10x_2 - x_3 = 8$

$x_1 + x_2 + x_4 = 7$

$x_1, x_2, x_3, x_4 \geq 0$ Ans: x_1^*=0, x_2^*=$\frac{4}{5}$, x_3^* = 0, x_4^* = $\frac{1}{5}$ Z_{min} =$\frac{4}{5}$]

(iii) Minimize $Z = \frac{15}{2}x_1 - 3x_2$

Subject to $3x_1 - x_2 - x_3 \geq 3$

$x_1 - x_2 + x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$ [Ans: x_1^*=$\frac{5}{4}$, x_2^*=0, x_3^* = $\frac{3}{4}$, Z_{min} =$\frac{75}{8}$]

28. Solve the following LPP, using two-phases of the simplex method:

(i) Minimize $Z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \quad\quad \leq 3$

$x_2 + 3x_3 \leq 5$

$x_1, x_2, x_3, \geq 0$ [Ans: x_1^*=$\frac{23}{23}$, x_2^*=5, x_3^* = 0, Z_{min} =$\frac{85}{3}$]

(ii) Minimize $Z = 2x_1 - x_2 + 2x_3$

Subject to $x_1 + x_2 - 3x_3 \geq 8$

$4x_1 - x_2 + x_3 \geq 2$

$2x_1 + 3x_2 - x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$ [The problem has an unbounded solution]

29. Solve the following LPP:

(i) Minimize $Z = 4x_1 + 2x_2$

Subject to $3x_1 + x_2 \geq 27$

$-x_1 - x_2 \leq 2$

$x_1 + 2x_2 \geq 30$

x_1 and x_2 are unrestricted in sign. [Ans: x_1^*=$\frac{24}{5}$, x_2^*=$\frac{63}{5}$, Z_{min} =$\frac{222}{5}$]

(ii) Minimize $Z = 2x_1 + 5x_2$

Subject to $x_1 + 2x_2 \leq 8$

$x_1 \leq 4$

$0 \leq x_2 \leq 3$

x_1 is unrestricted in sign. [Ans: x_1^*=2, x_2^*=3, Z_{min} =19]

30. an incomplete tableau of an LPP by simplx method is given bellow(intermediate stage).

		c_j	-4	-2	0	0	0	-M
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_{a_1}
-4	y_1	$\frac{24}{5}$			$-\frac{2}{5}$	0	$\frac{1}{5}$	
-M	y_{a_1}	$\frac{18}{5}$			$\frac{1}{5}$	- 1	$\frac{2}{5}$	
-2	y_2	$\frac{63}{5}$						
		$z_j - c_j$	0	0	$-\frac{M}{5}+\frac{6}{5}$	M	$-\frac{2M}{5}+\frac{2}{5}$	0

(i) Complete the table, (ii) Find the entering and departing vectors, (iii) Show that the unique optimal solution of the problem is $x_1^* = 3$, $x_2^* = 18$.