SERIAL NUMBER 1:

PROBABILITY

1. Show that for any two events A and B, $P(AB) \leq P(A) \leq P(A+B) \leq P(A)+P(B)$.

2. What is the probability that a leap year, selected at random, will contain 53 Sunday? $\left\{Ans : \dfrac{5}{36}\right\}$

3. In a bolt factory, machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output $5\%, 4\%, 2\%$ are defective bolts. A bolt is drawn at random from their product and is found to be defected. What are the probabilities that it was manufactured by machines A, B and C? $\left\{Ans : \dfrac{25}{69}, \dfrac{28}{69}, \dfrac{16}{69}\right\}$

4. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group. Show that the chance that the three selected children consisting of 1girl and 2 boys is $\dfrac{13}{32}$.

5. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ... , 98, 99. If X and Y denote the sum and product of the digits on the tickets respectively, find $P(X = \frac{9}{Y} = 0)$. $\left\{Ans : \dfrac{2}{9}\right\}$

6. An urn contains 3 white balls and 5 black balls. One ball is drawn and its colour is noted, kept aside and then another balls drawn. What is the probability that it is (i) black (ii) white? $\left\{Ans : \dfrac{3}{8}, \dfrac{5}{8}\right\}$

7. A box contains 5 defective and 10 non-defective lamps. Eight lamps are drawn at random in succession without replacement. What is the probability that the 8th lamp is the 5th defective? $\left\{Ans : \dfrac{5}{429}\right\}$

8. Show that the probability of occurrence of only one of the events A and B is $P(A) + P(B) - 2P(AB)$.

9. A and B in order throw alternatively with a pair of dice. A wins if he throws 8 before B throws 5 and B wins if he throws 5 before A throws 8. Find the probability that A wins. Assume that the game can continue indefinitely. $\left\{Ans : \dfrac{45}{76}\right\}$

10. A discrete random variable X has the following probability mass function:

Values of X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	k^2+k

(i) Find k. (ii) Evaluate $P(X < k), P(X \geq k)$ and $P(0 < X < 5)$.
$\left\{Ans : (i)\dfrac{1}{10}; (ii)\dfrac{81}{100}, \dfrac{19}{100}, \dfrac{4}{5}\right\}$

11. The probability density function is given by $f(x) = \begin{cases} kx^2, & 0 \le x \le 6; \\ k(12-x)^2, & 6 \le x \le 12; \\ 0, & \text{elsewhere.} \end{cases}$

(i) Evaluate the constant k. (ii) Find $P(6 \le x \le 9)$. $\left\{ Ans : (i)\dfrac{1}{144}, (ii)\dfrac{7}{16} \right\}$

12. A random variable X has the density function $f(x) = x, 0 \le x \le 1; f(x) = \frac{1}{2}, 1 < x \le 2$. Find the mean of X. $\left\{ Ans : \dfrac{13}{12} \right\}$

13. If a random variable X takes the values 1, 2, 3, and 4 such that 2P(X =1) = 3P(X =2) = P(X =3) = 5P(X =4), find the probability distribution of X.

$$\left\{ Ans : p_1 = \frac{15}{61}, p_2 = \frac{10}{61}, p_3 = \frac{30}{61}, p_4 = \frac{6}{61} \right\}$$

14. A random variable X has the following probability distribution:

X = x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

(i) Find k. (ii) Evaluate $P(X < 2), P(X \ge 2)$ and $P(2 < X < 2)$. (iii) Find the minimum value of k such that $P(X \le 2) \ge 0.8$ (iv) Determine the distribution function F(x) of X. $\left\{ Ans : (i)\dfrac{1}{15}, (ii)0.5, 0.5, 0.4, (iii)\dfrac{1}{15} \right\}$

15. A random variable X has the following p.d.f.: $f(x) = cx^2, 0 \le x \le 1$ and $f(x) = 0$, elsewhere. Find: (i) c (ii) $P(0 \le X \le \frac{1}{2}$. $\left\{ Ans : (i)3, (ii)\dfrac{1}{8} \right\}$

16. Show that the function f(x) given by
$f(x) = \begin{cases} x, & 0 \le x < 1; \\ k-x, & 1 \le x \le 2; \\ 0, & \text{elsewhere.} \end{cases}$ is a probability density function for a suitable value of the constant k. Construct the distribution function of the random variable X and compute the probability that the random variable X lies between $\frac{1}{2}$ and $\frac{3}{2}$. $\left\{ Ans : \dfrac{3}{4} \right\}$

17. The length of the life of a tyre manufactured by a company follows a continuous distribution given by the density function
$f(x) = \begin{cases} \frac{k}{x^3}, & 1000 \le x \le 1500; \\ 0, & \text{elsewhere.} \end{cases}$
Find k and find the probability that a randomly selected tyre would function for at least 1200 hours. $\left\{ Ans : 36 \times 10^5, \dfrac{9}{20} \right\}$

18. The p.d.f. of a random variable X is f(x) = k(x -1)(2 -x), for $1 \le x \le 2$. Determine (i) the value of k, (ii) the distribution function F(x), (iii) P($\frac{5}{4} \le x \le \frac{3}{2}$). $\left\{ Ans : (i)6, (iii)\dfrac{11}{32} \right\}$

19. The mean and s.d. of marks of 70 students were found to be 65 and 5.2 respectively. Later it was detected that the value 85 was recorded wrongly and therefore it was removed from the data set. Then find the mean and s.d. for the remaining 69 students. $\{Ans: 64.71, 4.64\}$

20. The probability density function of a continuous distribution is given by: $F(x) = \frac{3}{4}x(2-x), 0 < x < 2$. Compute mean and variance. $\left\{Ans: 1, \frac{1}{5}\right\}$

21. The radius of a circle has distribution given by the p.d.f.:
$F(x) = \begin{cases} 1, & 1 \leq x \leq 2; \\ 0, & \text{elsewhere.} \end{cases}$

    Find the mean and variance of the area of the circle. $\left\{Ans: \frac{7\pi}{3}, \frac{34}{45}, \pi^2\right\}$

22. The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates at least 4 passed examination? $\{Ans: 0.54432\}$

23. If the mean of a Binomial distribution is 3 and the variation is $\frac{3}{2}$, find the probability of obtaining atmost 3 successes. $\left\{Ans: \frac{21}{32}\right\}$

24. A random variable follows Binomial distribution with mean 4 and standard deviation $\sqrt{2}$. Find the probability of assuming non-zero value of the variable. $\left\{Ans: \frac{255}{256}\right\}$

25. If X is a Poisson variate with parameter $\lambda(> 0)$ then (i) mean $= E(X) = \lambda$ and (ii) var (X) $= \lambda$.

26. A car hire firm has two cars which it hires out day by day. The no. of demands for a car on each day is distributed as a Poisson distribution with average number of demand per day is 1.5. Calculate the proportion of days in a year on which neither car is used and the proportion of days on which some demand is refused. (Given: $e^{-1.5} = 0.2231$) $\{Ans: 81, 70\}$

27. If X follows normal distribution with mean 12 and variance 16. Find $P(X \geq 20)$ where $P(Z < 2) = 0.977725$. $\{Ans: 0.022275\}$

28. If X is normally distributed with zero mean and unit variance, find the expectation of x^2.

29. In a normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. (Given: $P(0 < Z < 1.405) = 0.496), P(-0.496 < Z < 0) = 0.19)$. $\{Ans: 49.958, 9.995\}$

30. The mean weight of 500 male students at a certain college is 150 lbs and the standard deviation is 15 lbs. Assuming that the weights are normally distributed, find how many students weight (i) between 120 and 155 lbs, (ii) more than 155 lbs? $(\psi(2) = 0.9772, \psi(0.33) = 0.6293)$. $\{Ans: (i)11, (ii)185\}$